The Calculation and Design of Ducted Fans

It is generally recognised that electrically-powered model aeroplanes distinguish themselves by their poor power-to-weight ratio, i.e. relatively little power, but lots of weight. There is no way around the fact that the energy which can be contained within the common Ni-Cd battery or the newer Ni-MH cells, expressed as Joule/kg, is less than that stored in the same weight of liquid fuel, such as methyl alcohol, petrol- or diesel fuel. In plain numbers ~135,000 Joule/kg in a Ni-Cd battery – or approx. 250,000 in case of the best Ni-MH cells - versus ~20,200,000 Joule/kg in methanol; that is about 80 to150 times as much energy per kg as in a battery! (The recent developments in the lithium battery field has improved the situation by another factor of two and more can justifiably be expected within the next years- at a price.) However the relationship improves when one considers that our excellent modern electric motors have a much higher efficiency when compared to any rattletrap combustion engine. An electric motor has an efficiency of about 80%, a combustion motor perhaps 10%, and that is on a good day. Despite this advantage, this still works out to odds of about 1:10 to18 in favour of the noisy IC engines.. In calculating these figures I have assumed, to some degree justifiably , that the drive trains, i.e. motors and "fuels" themselves have a similar power to weight ratio.

In absolute figures, however, the power requirements of our little aeroplanes are surprisingly small; theoretically (in a perfect world) the power required by an aircraft of whatever size depends solely upon the weight, angle of glide and airspeed. Because the angle of glide can also be expressed as the relationship between drag and lift; and the lift in steady level flight is equal to the weight of the aircraft, the power requirement can easily be expressed mathematically thus:

$$P_{flight} = W * a * D/L * v$$

The weight of the plane in kg (strictly speaking mass) has to be converted into a weight-force by multiplying with the gravitational acceleration (a=9.81 m/s^2). By shortening the right side term we get the very simple formula:

$$P_{flight} = D * v \text{ [Watt]}$$

Even simpler is the relation in respect of ducted fan (or jet) driven aeroplanes if we compare the thrust and the Drag over the flying speed. Then the following equation

$$D = T$$

must be true. Otherwise the plane would accelerate or retard until equilibrium is re-established or the plane impacts on terra firma.

N.B.
Denotations used for the physical properties often cause confusion. Here follows an explanation of the denotations used here, in accordance with international convention:

Power	*P*	*usually given in W [Watt]*
Weight	*W*	*usually given g [gram] or kg [kilogram]*
Gravitational acceleration	*a*	*9.81 m/s^2*
Weight-force	*W a*	*N [Newton]*
Drag	*D*	*always given in N [Newton]*
Lift	*L*	*always given in N [Newton]*
Velocity	*w or v*	*(general assignation usually given in m/s)*
Thrust	*T*	*always given in N [Newton]*

Other denotations are explained in the text, as necessary.

Sample calculation 1

To illustrate the usefulness of the previous reflections here a little example:

Model weight	1.0 kg (9.81N)
Glide ratio at v =15m/s	1/7
Thrust of EDF at v =15m/s	1.4N (143g)
Drag at v =15m/s	D = W * a * 1/7 = 1.4N
Required power	P_{flight} = W * a * D/L * v =1.0 * 9.81 * 1/7 * 15 = 21.0 Watt**

Perhaps some readers may recognise this as the notorious case of an early EDF model of a well known manufacturer of Styrofoam models.
One can see that at a flying speed of about 15 m/s the thrust is equal to the drag. Therefore there is no power remaining for further acceleration or climbing, due to the miserable inefficiency of the impeller and its installation at this speed.
At a wing loading of approx. 65g/dm² and the chosen configuration (swept back wing and profile) the minimum speed is already about 10-12m/s. So the possible flying speed range is very narrow indeed and controlling the plane becomes a bit of a struggle.
If however one compares the actually required power of only 21 Watt with the manufacturers quoted electrical input of about 130 Watt one can't help to see an obvious discrepancy which calls for an analysis. Efficiencies like the ones calculated from this example, i.e. 16%, shouldn't have a place in e-flight.

How do such mismatches occur? The case just quoted is a particularly extreme, though typical and quite recently repeated, example. I refuse to think that manufacturers set out to mislead their customers. Rather, I am convinced that ignorance of the influence of all the tiny alterations that are made to a model on its way from design to series production plays a greater role than intended.
So little concise and physically correct information about impellers, from basic principles to model building, has yet been published in a form accessible to every model builder and manufacturer that it can come as no surprise that a great many projects have ended in wasted efforts and disappointed customers.

So here we go, the basic principles of ducted fan theory and design made easy!

Or rather not?

It's a dry subject, and practically impossible to explain without using some rudimentary mathematics, as has been shown above. As we need to use quantities as well as qualities, words will not suffice. Model-building dentists be warned, the interested reader will just have to bite the mathematical bullet!
There may be some comfort in the knowledge that computer programs in form of several EXCEL spread sheets now exists that will calculate the model aeroplane's performance with variously altered parameters. It is suitable for all types of motors, but particularly for electrically driven ducted fans. With its aid anyone with access to a computer with Microsoft software installed can try out his own ideas about configurations without having to be a mathematical genius.

*** This is what is commonly called "absorbed" power. The supplied power must always be more because of the losses encountered in the supply chain.*

A little bit of simple physics to get to grips with

We shall now consider the grossly over-simplified 'idea' of a ducted fan, which will help us understand the principles on which it functions.
Fig. 1 shows a ducted fan, represented in its simplest form as an engine nacelle.

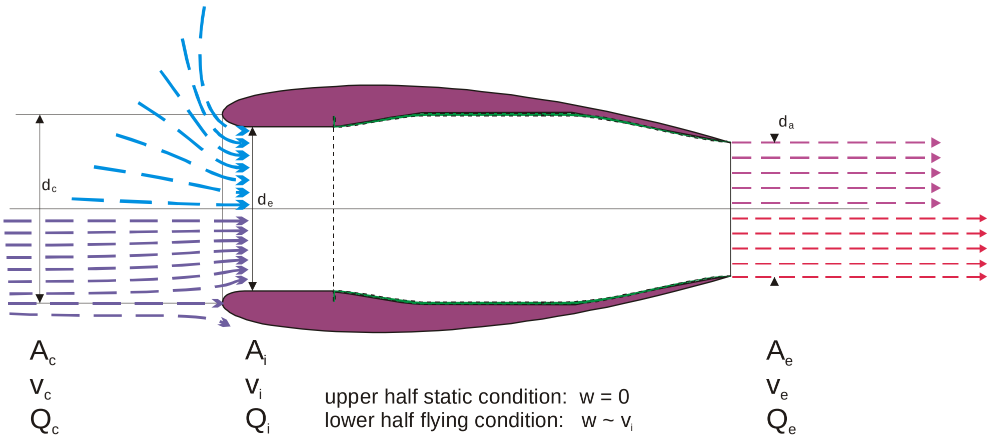


Fig. 1 Schematic illustration of a ducted fan nacelle with indicated streamlines of the air flow

When built into a model, the outer streamlined cover of the nacelle doesn't exist, and the air intakes and exits may be longer or even bifurcated. We shall not consider these details here for the moment, because the principle alone interests us, and that does not change. The complicated inside bits have been left out, quite deliberately, they have no part to play at the general considerations which interest us at the moment. Their time comes later.

The upper half of Fig.1 shows the ducted fan at rest, at a flying speed of w = 0. The lines around the intake represent the flow of air drawn into the duct evenly from all sides without flow detachment from the intake lip and a velocity v_i at the throat. The lines at the outlet represent the idealised stream of air as it leaves the nozzle with an increased speed v_e.
The lower part of Fig. 1 shows the impeller in flight at a speed $w \approx v_i$.
The even flow of air into the intake is represented by the nearly straight lines, and the increased speed of the out flowing air compared to the static case is shown by the somewhat longer lines (rather like a vectorial drawing). Strictly speaking, this illustration is not accurate, because the certainly existing turbulence in the out flowing air is not shown and the lower speed towards the duct wall from the centre, nor the possible flow detachments and unevenness at the intake.

Despite this over-simplification, one can recognise a principle at work here, which should be clear to everyone:

<u>Whatever air goes in at the front, has to come out at the back</u>

However one also should observe that the amount of air involved at the static condition must not necessarily be the same as in the flying state.

The amount of air "Q" referred to here is actually the volume of air flowing through the duct in every one second. One has to distinguish this from the mass flow –a term commonly used in full size aero engine design- which is the volume per second multiplied by the air's mass density.

In the more specialised literature one often finds a line or dot above the “Q” for the volume flow per second, which will be omitted in this text.

The equation

$$Q_i = Q_e \qquad [1]$$

manifests the law of continuity formulated by Bernoulli, a Swiss mathematician and physicist (1700 – 1782) who we will meet here again on various occasions.

One can easily visualise the amount of flowing air per second as a column of air, where the cross section area “A” is derived from the diameter under consideration and the length from the distance the air travels in one second, which represents the flow velocity “v”. Then we can write:

$$Q = A * v = d^2 \pi/4 * v \qquad [2]$$

where “d” is to be measured in Meter and “v” in Meter/Second (SI engineering units).

Combining [1] and [2] we derive:

$$d_i^2 \pi/4 * v_i = d_e^2 \pi/4 * v_e \qquad [3]$$

This shows that the inlet velocity and the outlet velocity of the air at the ducted fan shown in Fig.1 behave inversely proportional to the related cross section areas or the squares of their respective diameters.

Example:

A nacelle as drawn above has an inlet diameter d_i = 70mm and an exit nozzle diameter d_e = 60mm. Flying speed is assumed to be w = 30m/s so v_i is taken as 30m/s as well. The efflux velocity of the air leaving the nacelle then calculates to:

$$v_e = v_i * d_i^2 / d_e^2 \qquad [4]$$

$$= 30\text{m/s} * 70^2/60^2 \sim 41\text{m/s}$$

Whilst calculating the ratio of the diameter squared it is of no consequence using either mm or meter as units, the result is the same. In the example above it’s approximately 1.36.
By the way: in all our calculations we should try not to use too many decimals after the point so as not to imply an accuracy which is not reflected in reality.

For the example above we have established that the exit air velocity is ca’ 11 m/s faster than the velocity of the air entering the duct. This velocity difference we will call Δv (from the Greek Delta) in our following sample calculations; and it is this <u>velocity difference</u>, which is largely responsible for the development of thrust and also for the efficiency, which can be achieved with ducted fan engines.

Let’s now try to record, by calculating, the thrust of a ducted fan and its behaviour at various flying speeds according to the previously treated physical values like air volume Q, inlet velocity v_i and outlet velocity v_e.

Calculating the Thrust

The working principle of a ducted fan, as we have seen already further up, is in fact quite simple: air enters at the front end and is expelled at the rear with an increased velocity. That means air is accelerated and generates as reaction to this acceleration process a force, which we can measure as a propulsive force and call thrust. According to Newton's third fundamental law of mechanics this process can be described by a simple equation:

$$T = M * \Delta v \qquad [5]$$

Or put in simple words for our purpose: Thrust (T) equals Mass flow (or mass throughput per second) times velocity difference (Δ v). There should be a dot or a little line above the M, but see the explanation for "Q" further above.

The mass flow simply calculates from the air flow "Q" as defined previously multiplied by the mass density of the prevalent air.

Now one could argue about the value of the air density, which varies with temperature, humidity and naturally the height at which it's measured. In all calculated examples here the air density ρ (Greek lower case Rho) is taken as $1.2kg/m^3$. This is a value which is commonly measured in our latitudes at sea level, is easy to remember and to calculate without too many decimals (see remark above).

This completes our equation [5] to:

$$T = Q * 1.2 * \Delta v \qquad [6]$$

where the thrust is calculated in Newton (N) if one uses the already defined SI units.

For our above started example the values are as follows:

$T = d_i^2 \, \pi/4 * v_I * 1.2 * \Delta v = 0.070^2 \, \pi/4 * 30 * 1.2 * 11 = 1.52N$ (or ca' 150 Gram)

We see that our little ducted fan still delivers approx. 150gram (1/4+ lb) of thrust at a flying speed of ca' 30m/s (108km/h or ~ 67mph). This really is not exhilaratingly much. However, if one considers that a reasonably well designed and built model at this speed can have a drag to lift ratio of 1/9, it is still sufficient to get a 1kg plane up to this velocity.

But how could one increase the thrust at the mentioned flying speed?
The solution according to equation [6] shows two possibilities: either increase "Q" (which requires a larger inlet diameter) or Δv (which would call for a smaller outlet diameter).
Or perhaps both?

The ducted fan in Fig.1 actually reveals the correct way to go. If one looks carefully at the front end one can distinguish the rounding of the inlet lip and the marked diameter d_c, which refers to the front most circle. Corresponding to the stagnation point at the nose of an airfoil the air flow divides here and flows either into the duct or outside around the engine nacelle. The whole has to be visualised in a three dimensional way i.e. symmetrically rotated around the centre axis. The cross section area thus derived from d_c could also be called the "catchment" area.
Fig. 2 shows an example for a new technology engine nacelle, the BR 715, as used on the Grumman Gulfstream. It indicates as to how an efficient air intake should look. One can also see quite clearly that the inlet is substantially larger than the exit nozzle diameter.

Fig 2

Inasmuch as the effective inlet diameter is slightly enlarged by the rounding off of the inlet lip, the effective air flow and the differential velocity between inlet and outlet is increased.

Let's assume the rounding of the inlet lip in the sample calculation to be 2.5mm, then d_c is 75mm. The exact shape of the lip round-off whether elliptical, circular, parabolic or any other combination shouldn't interest us here yet.

The net effect of the rounding of the intake lip is a significant increase of the air flow through the unit at the same flying speed. In our example at a flying speed of w = 30m/s the air flow is increased by the factor $d_c^2 / d_i^2 = 75^2 / 70^2 \sim 1.15$. By this measure not only the air flow is influenced positively but also the exit velocity at the nozzle, since, because of the continuity law, the air is already "pre-accelerated" by the same factor when it enters the inlet cross section A_i. This means that the exit velocity also grows from ca' 41m/s to ~47m/s. A new calculation according to equation [6] then renders:

$T = 0{,}075^2\ \pi/4 * 30 * 1{,}2 * 17 \sim 2.7$ N.

Surprisingly we have increased the thrust <u>potential</u> at some set flying speed by more than 77% by merely rounding the intake lip with a radius of 2.5mm!

<u>This little example of simple arithmetic shows very clearly how important it is to pay attention to correctly designing the intake of a ducted fan.</u>

Naturally one can not carry on indefinitely with the miraculous proliferation of the thrust force of a particular ducted fan, since the acceleration of the air stream has to be paid for by a considerable expenditure of power. And as so often in real life one has to pay over the odds in power to achieve a relatively small increase in acceleration.

So obviously the absolute cross section of the inlet S_i (and the shape of the inlet lip) for the air flow involved and the ratio between the inlet- cross section and outlet cross section for the acceleration of the air flow plays a major role in designing a ducted fan.
How much of each is "just right" and therefore most economical we can only judge further on in this article, when insights into the mechanics have become a little bit clearer. Here we will first carry on with a more pressing problem.

The damn lost power and the miraculous regain

As already mentioned above the acceleration of the air flow requires power.
This "used" power is however found again in the air stream as so called "impulse loss" and can be expressed as the product of half of the air mass flow per second multiplied by the square of the nozzle velocity. This mouth full is easier written as a vivid equation:

$$I_{loss} = P_{loss} = M / 2 * v_e^2 \qquad [7]$$

Expressed in physical denominations this produces for our example calculation:

$Q = d_c^2 \, \pi/4 * w = 0{,}075^2 \, \pi/4 * 30 = 0{,}133 \text{ m}^3/\text{s}$, hence

$M = Q * \rho = 0{,}133 * 1{,}2 = 0{,}16 \text{ kg/s}$ and

$v_e^2 = 47^2 = 2209 \text{ m}^2/\text{s}^2$:

$I_{loss} = P_{loss} = M / 2 * v_e^2 = 0{,}16 \text{ kg/s} / 2 * 2209 \text{ m}^2/\text{s}^2 = 177 \text{ Watt}$ *

Now, our ducted fan (built into or at the outside of a model aeroplane) moves itself with the flying speed w = 30 m/s through the air and therefore experiences a head wind of the same magnitude. According to this head wind something happens what most modellers (and even some DF – "Gurus") think is non existing, unimportant and negligible. In reality the effect is measurable, very important and sometimes even decisive in respect of the success or failure of a ducted fan model. Certainly the effect must be included if one considers the design of these machines.

It's the impulse or energy regain at the intake face we are talking of here.

Did we previously consider the impulse losses of the out-flowing air stream, according to the same principles we can justifiably assume that the air stream entering the inlet contains a certain amount of energy relative to the plane when this moves through the air. At rest when air is sucked into the duct there will naturally no gain.

Analogue to equation [7] we can therefore write:

$$I_{gain} = P_{gain} = M / 2 * w^2 \qquad [8]$$

- *Author's remark: the above dimensional consideration supplies as end result kg/s x m^2/s^2 = kg m^2 / s^3. In the considered case we have a mass flow and not a weight flow so that the result should be divided by the earth's gravitational acceleration = 9.81 m/s^2 to obtain the correct dimension Nm/s for the Watt. Since 1 kgm/s = 9.81 Watt one normally saves the transformation because it results in the same numerical value. The deeper reason for this dimensional discrepancy derives from the habit in the calculation of air forces to use the air density p (Rho) with 1.2.. kg/m^3 to achieve a numerical result in N (Newton). Strictly speaking one would have to divide the density by 9.81 m/s^2 to achieve "mass density" The correct dimension for p (Rho) therefore is actually 1.2.. kg/m^3 / m/s^2 = kg s^2 /m^4, with this the dimensional considerations in respect of the lost energy are again correct.*

For our sample ducted fan the picture with the assumed values then is as follows:

M = 0.16 kg/s and

$w^2 = 30^2 = 900\ m^2/s^2$

and a regain of P_{gain} = 0.16 / 2 * 900 = 72 Watt

This is 40% of the required fan power, which in my opinion cannot be called negligible!

Let's summarise then the marked values of the example as far as they have been fixed:

Inlet diameter d_i = 70 mm, 2.5mm inlet lip rounding, i.e. d_c = 75 mm
Exit nozzle diameter d_e = 60 mm
Flying speed w = 30 m/s
Air flow Q = 0.13 m^3/s (at w = 30 m/s)
Mass flow M = 0.16 kg/s
Thrust at 30 m/s = 2.7 N
"Lost" power = 177 W "Power gain" (at w = 30 m/s) = 72 W

If one now subtracts the power gain from the lost power one calculates the power which has to be transferred by the fan to the air stream so that it can produce the a.m. thrust at the appropriate velocity, therefore:

$$P_{fan} = P_{loss} - P_{gain} \qquad [9]$$

And in our case:

P_{fan} = 177 – 72 = 105 W

Against this stands the power, which the propulsion unit transfers to the model (or the model "absorbs"), in as far as it propels the model at a steady speed and overcomes (equals) it's drag. At the very beginning of this chapter we had already a look at this without numbering the appropriate equation, so here it is again:

$$P_{flight} = D * w = T * w \qquad [10]$$

That means that the power absorbed by the (model) airplane at a steady flying speed is equal to the product of the thrust (T) times velocity (w).

In our example we get:
P_{flight} = T * w = 2.7 N * 30 m/s = 81 W

The Ducted Fan and its Efficiency

Here now the dilemma of the EDF mania becomes blatantly obvious (not quite as bad as in our initial example but nevertheless):

The power which is transferred to the air stream by the fan and the used power to propel the plane are far apart. This unchangeable fact has to be viewed with bravery by the electrically minded ducted fan enthusiasts and they have to learn to live with it. Until someone comes along and redefines currently accepted basic physics we have to strive to minimise the inevitable loss to it's lowest by clever design.

The ratio between the two powers P_{flight} and P_{fan} is usually called the ideal or external propulsion efficiency. In the Anglo-Saxon literature it is often described as the Froude efficiency after the famous British physicist William Froude (1810 – 1871)

Let's put the above statement into an equation:

Propulsion efficiency $\eta_{propulsion} = P_{flight} / P_{fan}$ [11]

Expressed in numbers we calculate for our ducted fan:

$\eta_{propulsion}$ = 81 / 105 (Watt) = 0.77 or 77%

And this is only the Froude efficiency. This still has to be multiplied by the "internal" efficiency and the electrical power train efficiency (each approximately 0.8 if we are lucky) to receive the overall efficiency of our drive system. We'll talk about that a little bit later.

Absolutely shocking figures tend to appear –if we are honest- which actually should deter any sane modeller from getting involved with electrically driven ducted fans. By the way these inescapable facts have led to the abandonment of ducted fans as a means of propulsion in the real world of aviation – after the initial enthusiasm - for low powered and low speed applications. In the 1970's there were some interesting developments –the Fan Islander by Britton-Norman and the Fantrainer spring to mind. However in this field normal propellers are definitely at an advantage because of their higher efficiency. When it comes to the huge, powerful fan engines of commercial and military transport aviation the situation is different because of the very high powers and flying speeds involved.

But for us modellers the ducted fan propulsion system, especially the electric one, is only a means to serve a special purpose, namely to drive model aeroplanes which resemble real or nearly real "jet" aeroplanes. And since the efficiency of the drive system only plays a minor part in its economical sense (we don't have to transport a payload over a given distance at a given cost – and make a profit) we can play around with solutions which work for our purpose and enjoy the solutions which do work.

The above calculated sample ducted fan is in its dimensions, as far as they have been established, by no means a chance product, it's rather a carefully chosen object. The influences of other dimensions at other possible design criteria can easily be determined by inputting the appropriate data in the accompanying PC program. This program allows you - the reader- to play around with the variables and see how they influence the performance.

However, now we shall see how our ducted fan behaves at other flying speeds than the above assumed 30 m/s.

The famous – infamous Static Thrust

For this we have to resolve the calculation problem of a ducted fan at the flying speed of w = 0 m/s.

This condition is shown in the upper half of Fig. 1. According to our previous considerations one can say with surety that the lost power must be calculated again according to equation [7] and that a power gain according to equation [8] is non existent, since the flying speed is zero. To recap, the equation is once more given here to avoid paging back:

$I_{loss} = P_{loss} = M / 2 \times v_e^2$

The awkward situation however is that we have to realise by looking at this equation, that practically all values are unknown and hence the equation seems to be useless! We therefore have to use a legitimate trick and make the assumption that the fan power is the same as in the previous case at w = 30 m/s, ergo 105 W in our example.. This assumption is insofar justified as one can expect that the (fan) power which is available in flight and is used to accelerate the air flow then, is also available in the static condition. Naturally the applied shaft power is higher than the fan power of the air stream since the fan itself is not 100% efficient but these losses will be dealt with in their turn.

Geometrically there is no change of the ducted fan between the considered states so that there must be the same relations as in the flight condition.

Now according to equation [6] the thrust is calculated as T = Q x 1.2 x Δv,
and Δv is now equal to the exit velocity v_e of the air stream at the nozzle since the air stream has been accelerated from 0 to v_e. Let's call the static thrust (as a special case) T_0 and also give the index 0 to all other related values then we can rewrite equation [6]:

$$T_0 = Q_{e0} * 1.2 * v_{e0} \qquad [12]$$

The airflow Q is, as we have seen beforehand, dependant on the velocity and the diameter according to equation [2] and therefore here:

$$Q_{e0} = d_e^2\ \pi/4 * v_{e0}$$

Inserted in equation [12] we get:

$$T_0 = d_e^2\ \pi/4 * v_{e0} * 1{,}2 * v_{e0} = 1{,}2 \text{ x } \pi/4 * d_e^2 * v_{e0}^2 = 0{,}9425 * d_e^2 * v_{e0}^2$$

Leaves as only unknown still v_{e0}, which we get after expansion and rearrangement from the lost energy equation [7] :

$$P_{loss.} = M / 2 * v_{e0}^2 = (1{,}2\ d_e^2\ \pi/4 * v_{e0} / 2) * v_{e0}^2 = 0{,}471 * d_e^2 * v_{e0}^3$$

And following:

$$v_{e0}^3 = P_{loss} / 0{,}471 * d_e^2 \quad \text{and}$$

$$v_{e0} = \sqrt[3]{\frac{Pl_{loss}}{0{,}471 \text{ x } d_a^2}} \qquad [13]$$

Since we have got so far we should quickly calculate the exit velocity of the air for the static case of the sample ducted fan:

$$v_{e0} = \sqrt[3]{\frac{105}{0{,}471 \text{ x } 0{,}060^2}} = 39.5 \text{ m/s}$$

Surprisingly or not, this is quite a lot (in this example 19%) less than the calculated air exit velocity for the case at flying speed of w = 30m/sec.
This is under the condition that the power output of the motor and the internal losses can be assumed to be equal.
Discounting those minor deviations, the fact manifests itself here that the exit velocity of the air stream v_e is least at the static case and increases with flying speed (for one and the same DF-unit and the same power setting but not at all the same motor rotational speed).

In the mean time we can insert v_{e0} in equation [12] and get after some multiplications and rearranging:

$$T_0 = 1.52 * d_e^2 * (P_{loss.} / d_e^2)^{2/3} \qquad [14]$$

(The factor 1.52 at the beginning is a rounded figure in line with an endeavour to reduce the digits after the decimal point of a number. The accurate value lies between 1.471 and 1.558

dependent on the air density and therefore varies with barometric pressure, temperature and humidity.)

I could have introduced this revealing and informative “formula” here without detailed derivation but decided against this despite my being aware that many model builders and flyers have aversions towards maths.
With the above given derivation however one can understand that the result has a logical background and is only based on the strict application of basic physics.
This formula, which was first published 1982 in an article which I wrote for MFI -a German model magazine- is an extremely important key for all ducted fan calculations.
It affords by the variation of only two variables –the exit jet diameter and the lost power- to calculate the expectable static thrust of a given design. Form and type of the power supply play no part at this early stage.

Let’s see what this equation can contribute to our sample ducted fan:
The exit diameter was fixed at d_e = 60 mm and the lost power or fan power P_{fan} with 105W. Inserted in equation [14] results in:

$T_0 = 1{,}52 \times 0{,}060^2 \times (105 / 0{,}060^2)^{2/3} = 1{,}52 \times 0{,}003600 \times (105 / 0{,}003600)^{0{,}666} = 5.2$ N = 530 gr or ca’ 19oz

(As an exception we had to use multiple digits after the decimal point here because of the small numbers involved. Otherwise accuracy would suffer. The pocket calculator makes it possible.)

For those who use a PC the provided program can be used to evaluate the wildest phantasies and also whether the purchased EDF unit comes up to claims after measurements are made.

Even more Efficiencies

The burning question now is how the lost power relates to the motor power so that we can use the real thing instead of the imaginary air stream power which really doesn’t interest us very much in our further considerations and calculations. What interests us is a simple method to determine the thrust on the basis of the physical dimensions and the motor power.

The ratio of the fan power (which is the same as the lost power in relation to the air flow) to the motor power of an air moving machine can be designated the internal efficiency of such a device i.e. $\eta_i = P_{Fan} / P_{Mot}$.

The losses inside a ducted fan are composed of three main constituents: 1. the fan itself, *i.e.* losses of the blading of rotor and stator, 2. the conversion of pressure into velocity and vice versa and 3. the friction of the air stream on the inside walls of the fan and ducting housing. Without further proof –which comes later under the heading of pipe losses- we can neglect the friction losses of 3. here for the time being. Those losses are negligible for a ducted fan of the type shown in fig. 1 and 2. As a rule they are usually only a few percentage points of the installed power.

The losses of the fan rotor and stator can be quite substantial if the geometry is not appropriate, but with a correctly engineered design – which is the aim of this exercise and which will be explained here in depth - the losses are of the order of between 15% and 8% , where the latter value has to be seen as the lowest limit which may be achieved with our means.

Remains the second source, as indicated above, the transformation of velocity into pressure (flow through a diffuser) or conversely pressure into velocity as happens in a nozzle.

Both conversions are subject to losses where according to all experience (and physical laws) those of the flow through a diffuser are larger than those experienced with flow through a nozzle.

The minimal possible losses according to 1. and 2. are predominantly dependant on the ratio of the cross sections at the air entry and air exit i.e. the inlet (catchment) area and the jet nozzle.

They are shown in fig. 3.

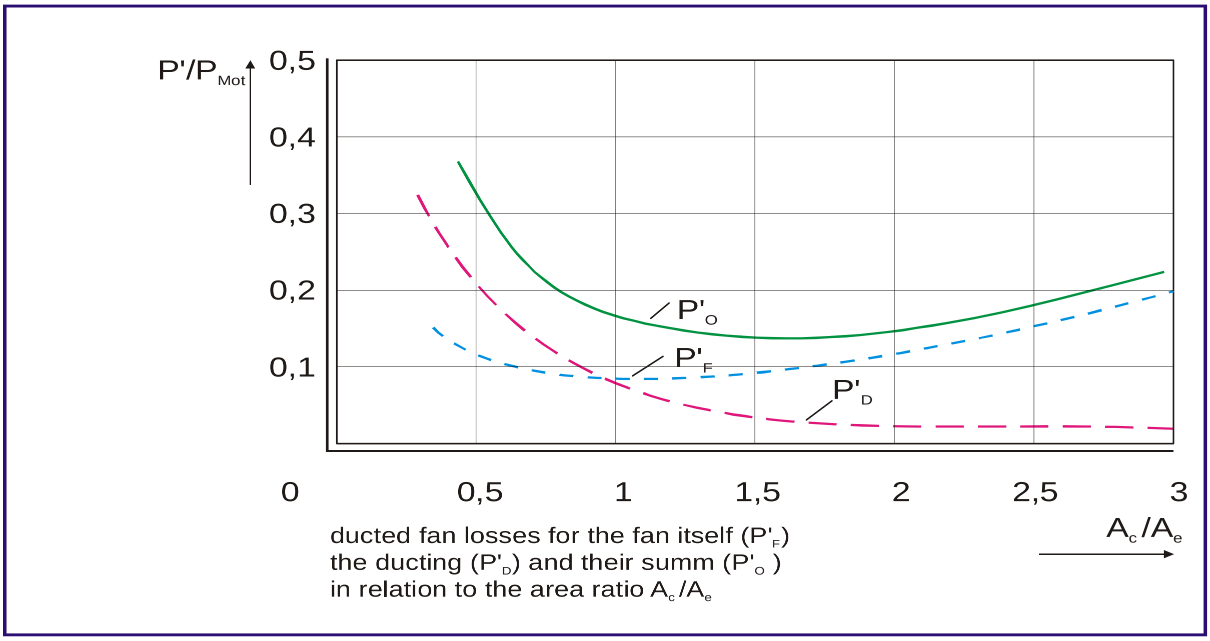


Fig 3

For the calculation of the curves I used ducted fan specific design criteria, general laws of air flow and some generally acknowledged empirical data for fan design. We will refer to the derivation of these curves at a later chapter of this treatise. Here the general tendency without claim to absolute accuracy should be represented. There may be deviations from the presented curves for individual cases which will however not influence the results in the sense as outlined here.

From the graph we can derive that the fan losses, here designated as P'_F are lowest if the area ratio A_c / A_e = 1 and increase only slightly if the ratio A_c / A_e increases. At area ratios smaller than 1, *i.e.* if the "catchment" area becomes smaller than the jet exit area the losses increase over proportionally. At A_c / A_e = 0,5 the combined fan losses rise above 30 %.
The curve of the "transforming losses" here called P'_D follows a hyperbolic form and falls over the viewed range from very high values at small area ratios to practically 0 at area ratios greater than 3.
Adding up the "transforming losses" and the fan losses the sum constitutes the "internal losses" of the ducted fan, which are designated P'_0 („o" for overall).
The curve of the total internal losses has it's lowest values of 15-18% at area ratios of between 1.2 and 2. This indicates that the ducted fan (generally) can achieve its best efficiency of approx. 82-85% at these conditions.
This graph also shows conclusively that the <u>catchment area A_c must be greater than the exit nozzle area A_e</u>, if reasonable efficiencies of the EDF (small internal losses) are to be achieved.
A fundamental realisation many EDF modellers have found empirically by trial and error.

In the case of the EDF-model introduced at the beginning of this chapter this basic principle was not observed (probably for manufacturing reasons or perhaps because of plain ignorance) with the described devastating results.

Unfortunately the suggested improvements in the specialist magazines mainly advocated increased power installations according to the maxim: “in the end only more power helps”. Only one suggestion lead into the direction of changing the ratio A_c / A_e by virtue of the installation of a yoghurt pot with cut off bottom at the nozzle end, which was the simplest and cheapest method to get the plane into the air. The latter suggestion was published in EFI – the magazine for switched on modellers.

However - we can now finally answer the question for the drive power as mentioned above: since the internal losses of the ducted fan are of the order of 15% according to fig.3 (if the machine is designed and built properly) the achievable efficiency is then 85 % and $\eta_i = 0,85$ and therefore
$P_{Fan} = P_{mot} \times 0,85$.

So for the static thrust equation we can now write:

$$T_0 = 1,52 * d_e^2 * (P_{mot.} * 0,85 / d_e^2)^{2/3} \qquad [14.1],$$

And since the “lost power” of our sample calculation above is 105 Watt, the motor power (also called shaft power) calculates to 105 / 0.85 = 123 Watt.
This is around the 125W mark with which I had started my original calculations. The discrepancies of approx. 1% are the result of various figure roundings.
If we consider a motor efficiency of 0.6 (60%) we calculate an electrical input power of a very realistic 200 Watt which is just comfortably within the “Speed 600” motor range.

And so this EDF with a nozzle diameter of 60mm and a nicely rounded inlet of 70mm diameter is just the thing for this size of motor. Fan outer diameter will be 80mm and the hub 40mm - but I’m jumping ahead of myself.

I will later show that with more powerful motors (up to around 1200W) in the same housing and the identical fan static thrusts of up to 20N can be achieved.
This is naturally not applicable if the EDF is installed within a fuselage with long inlet and outlet ducting. In such arrangements at the top end of the motorisation, duct losses will be high and can reach around 10% to 15% of the gross fan thrust.

The figure 4 below gives an indication of the performance of such an advanced fan.
It shows also that above certain power levels (in this case perhaps 1.5kW) further increases of the fan rotational speed are barely possible because the power requirement rises with the cube of the fan revs. A typical case of diminishing returns.

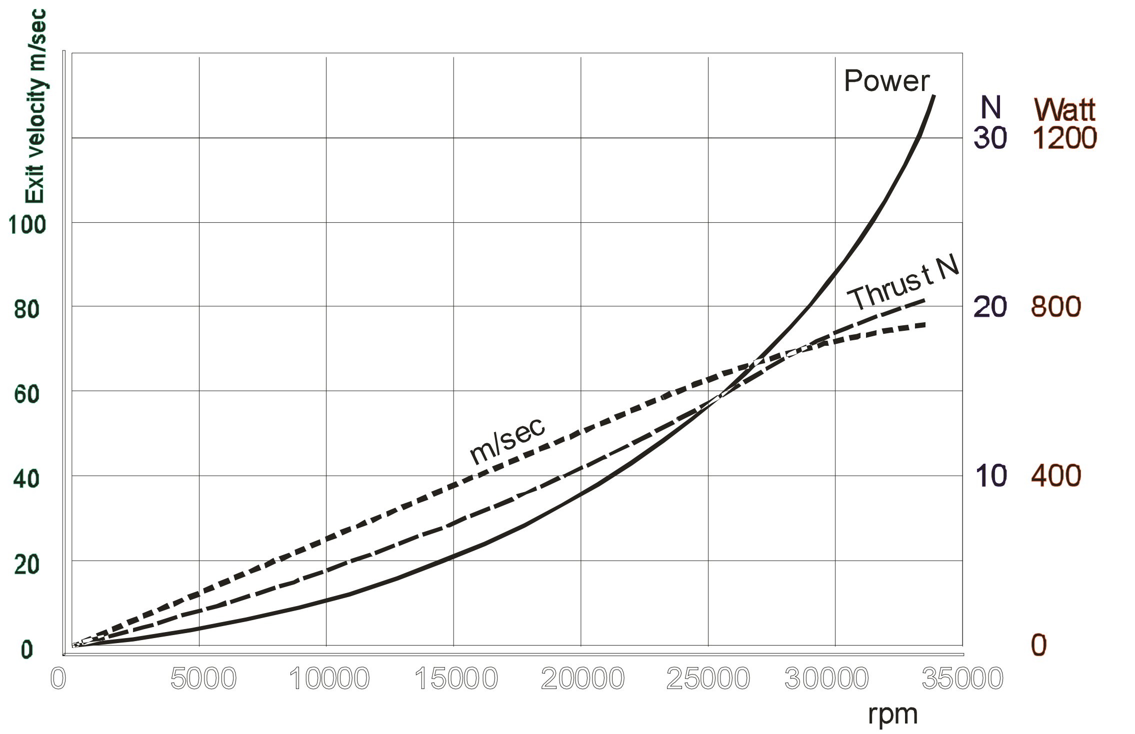
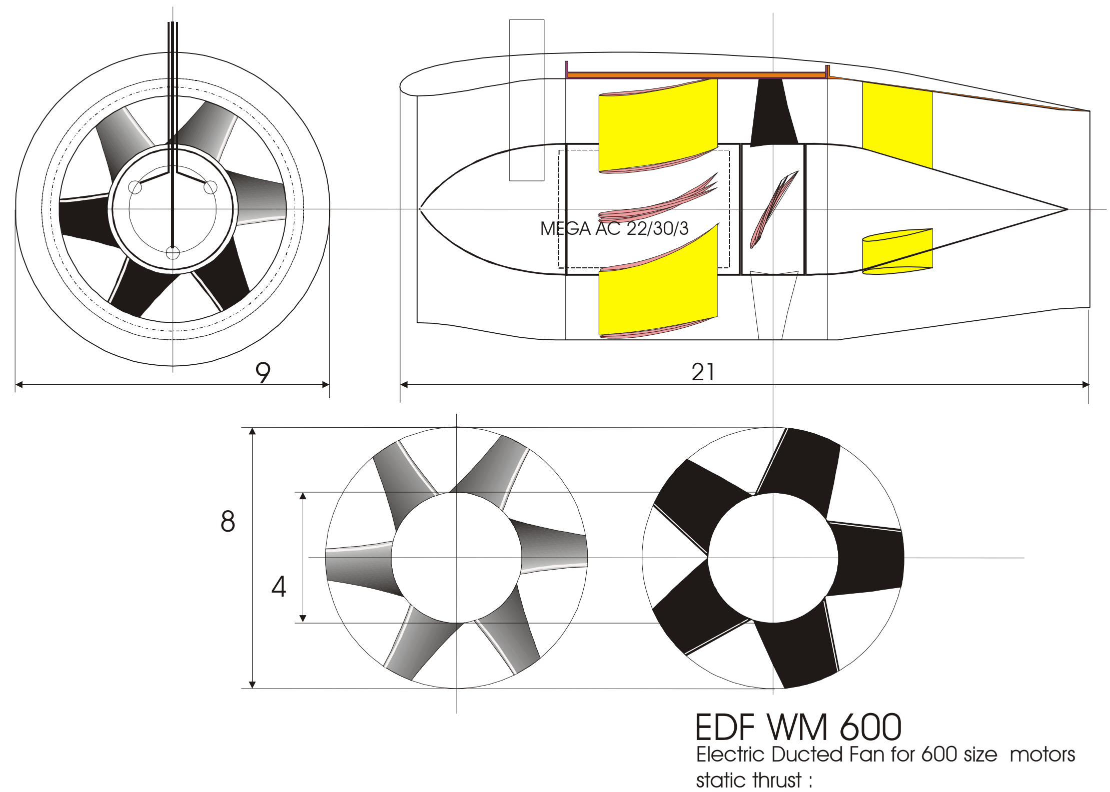


Fig.4



Preliminary sketch II Scale 1:1

Here we can see how the sample EDF looks.

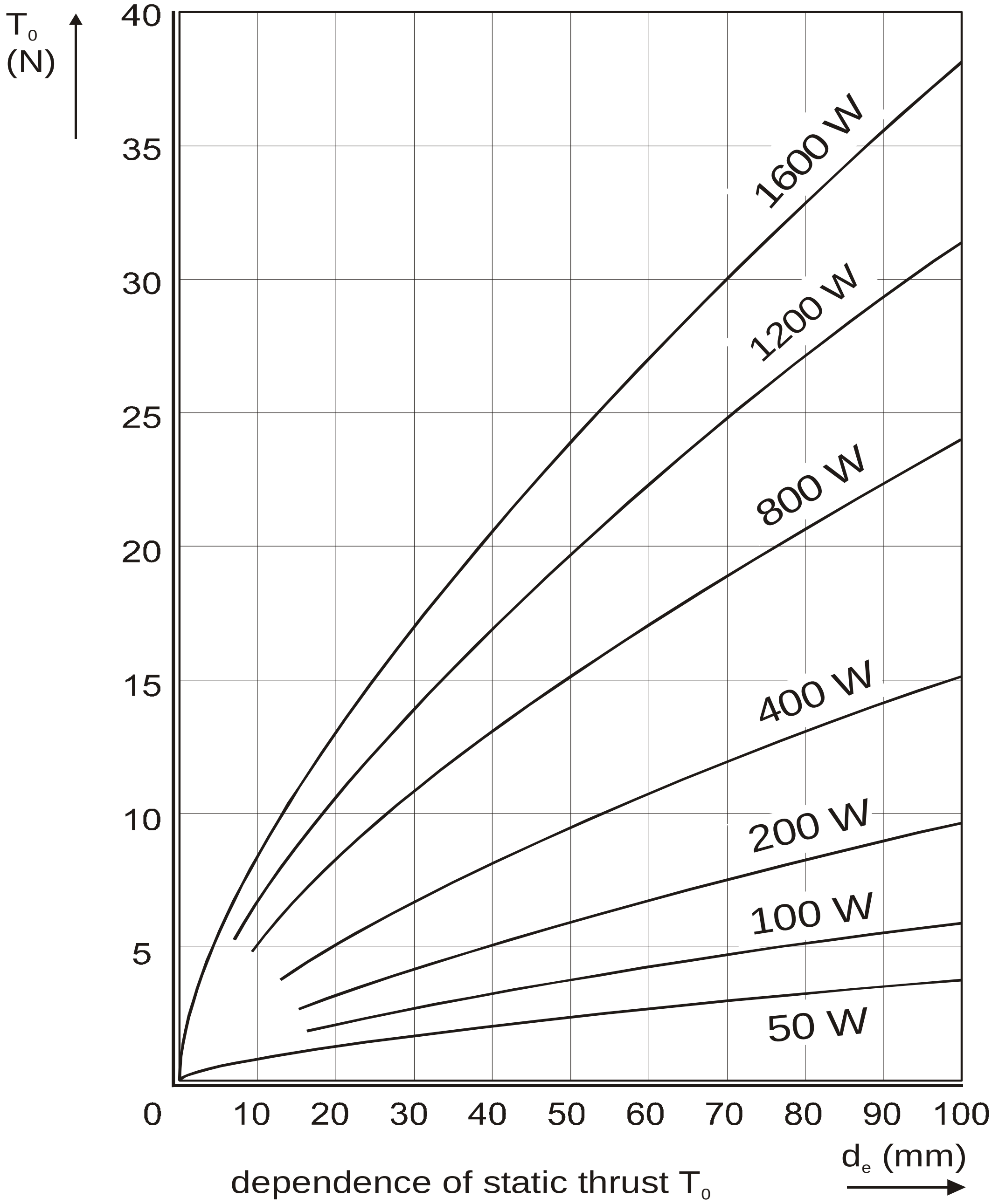


Abb. 6 dependence of static thrust T_0 on the nozzle diameter d_e for various power inputs

In Fig. 6 I have shown the results derived from equation [14.1]. Here we can see several curves starting at the zero point which show the relation of the static thrust development with increasing jet nozzle diameter for various power levels. As can be expected low power levels lead to small static thrust figures; but what is surprising is the realisation that the thrust for increasing nozzle diameters only increases slowly at these power levels, i.e. the curves run very flat. However when we have lots of power –say above 400W the static thrust gain is already considerable at rather small enlargement of the nozzle exit diameter. At real high power levels it pays to chose as large an exit diameter as can be accommodated in the model (which are probably still smaller than the dimensions seen on currently available EDFs –but that's another story).
At power levels past the 1000 Watt a small increment in the exit nozzle diameter results in large thrust increases. However, doubling the motor power in the same ducted fan will yield not more than a 63% increase in static thrust.

The Calculation and Design of Ducted Fans

It is generally recognised that electrically-powered model aeroplanes distinguish themselves by their poor power-to-weight ratio, i.e. relatively little power, but lots of weight. There is no way around the fact that the energy which can be contained within the common Ni-Cd battery or the newer Ni-MH cells, expressed as Joule/kg, is less than that stored in the same weight of liquid fuel, such as methyl alcohol, petrol- or diesel fuel. In plain numbers ~135,000 Joule/kg in a Ni-Cd battery – or approx. 250,000 in case of the best Ni-MH cells - versus ~20,200,000 Joule/kg in methanol; that is about 80 to150 times as much energy per kg as in a battery! (The recent developments in the lithium battery field has improved the situation by another factor of two and more can justifiably be expected within the next years- at a price.) However the relationship improves when one considers that our excellent modern electric motors have a much higher efficiency when compared to any rattletrap combustion engine. An electric motor has an efficiency of about 80%, a combustion motor perhaps 10%, and that is on a good day. Despite this advantage, this still works out to odds of about 1:10 to18 in favour of the noisy IC engines.. In calculating these figures I have assumed, to some degree justifiably , that the drive trains, i.e. motors and "fuels" themselves have a similar power to weight ratio.

In absolute figures, however, the power requirements of our little aeroplanes are surprisingly small; theoretically (in a perfect world) the power required by an aircraft of whatever size depends solely upon the weight, angle of glide and airspeed. Because the angle of glide can also be expressed as the relationship between drag and lift; and the lift in steady level flight is equal to the weight of the aircraft, the power requirement can easily be expressed mathematically thus:

$$P_{flight} = W * a * D/L * v$$

The weight of the plane in kg (strictly speaking mass) has to be converted into a weight-force by multiplying with the gravitational acceleration (a=9.81 m/s^2). By shortening the right side term we get the very simple formula:

$$P_{flight} = D * v \text{ [Watt]}$$

Even simpler is the relation in respect of ducted fan (or jet) driven aeroplanes if we compare the thrust and the Drag over the flying speed. Then the following equation

$$D = T$$

must be true. Otherwise the plane would accelerate or retard until equilibrium is re-established or the plane impacts on terra firma.

N.B.
Denotations used for the physical properties often cause confusion. Here follows an explanation of the denotations used here, in accordance with international convention:

Power	*P*	*usually given in W [Watt]*
Weight	*W*	*usually given g [gram] or kg [kilogram]*
Gravitational acceleration	*a*	*9.81 m/s^2*
Weight-force	*W a*	*N [Newton]*
Drag	*D*	*always given in N [Newton]*
Lift	*L*	*always given in N [Newton]*
Velocity	*w or v*	*(general assignation usually given in m/s)*
Thrust	*T*	*always given in N [Newton]*

Other denotations are explained in the text, as necessary.